

Universality in the large N_c dynamics of flavour: thermal vs. quantum induced phase transitions

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ABSTRACT: We show how two important types of phase transition in large N_c gauge theory with fundamental flavours can be cast into the same classifying framework as the meson-melting phase transition. These are quantum fluctuation induced transitions in the presence of an external electric field, or a chemical potential for R-charge. The classifying framework involves the study of the local geometry of a special D-brane embedding, which seeds a self-similar spiral structure in the space of embeddings. The properties of this spiral, characterized by a pair of numbers, capture some key universal features of the transition. Computing these numbers for these non-thermal cases, we find that these transitions are in the same universality class as each other, but have different universal features from the thermal case. We present a natural generalization that yields new universality classes that may pertain to other types of transition.

KEYWORDS: Gauge-gravity correspondence, AdS-CFT Correspondence, D-branes.

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1. Introduction

A lot of attention has been focused on the properties of the system consisting of the intersection of N_c color Dp -branes and N_f flavour Dq -branes ($p < q$). In the large $N_c \gg N_f$ limit the Dp -branes can be substituted by their corresponding black p -brane supergravity background, while the Dq -branes are in the probe limit [1].

In addition the Dq -branes are extended along $q-p$ of the $9-p$ dimensions transverse to the Dp -brane, and as a result their gauge degrees of freedom are frozen compared to those of the Dp -brane. The dynamics of the $p-q$ strings (which transform in the fundamental of the $SU(N_c)$ low energy gauge theory) and the $q-q$ strings are described by the Dirac-Born-Infeld action of the Dq -branes.

Among the issues of interest was the study of the thermodynamic properties of the dual [2–4] Yang-Mills theory and certain thermal phase transitions in the dynamics of the fundamental matter. The first study of this nature was for the D3/D7 system and was considered in ref. [5], where the authors considered the near-horizon limit of the non-extremal black 3-brane solution, corresponding to the $AdS_5\text{-BH} \times S^5$ geometry (the anti-de Sitter (AdS) spacetime contains a black hole). The D7-brane wraps a $S^3 \subset S^5$ and extends in the radial direction of the $AdS_5\text{-BH}$. The size of the S^3 varies as a function of the radial coordinate. The D7-brane embeddings then naturally form two classes: embeddings that reach the horizon and hence fall into the black hole, and embeddings for which the wrapped S^3 shrinks to zero size at some radial position. For these, the D7-brane world-volume simply closes smoothly before the horizon. In an Euclidean presentation, the compact, unbounded parts of the D7-brane have the topology $S^3 \times S^1$ since the Euclidean time has a periodicity set by the inverse temperature of the system. The classes are then distinguished by one or the other compact space shrinking away. The authors of ref. [5] proposed that

the (topology changing) transition of the D7-brane embeddings corresponds to a type of confinement/deconfinement phase transition, now in the meson sector of the theory. This system has been extensively studied in refs. [8]–[27] and it was shown that it is a first order phase transition providing a holographic description of the meson melting phase transition of the fundamental matter.

There is also a unique critical embedding separating those two classes. This solution reaches the horizon *and* has a shrinking S^3 . It has a conical singularity. Solutions of this type will occupy much of our attention in this paper. Many of these features generalize to the general Dp/Dq system. In ref. [9] the Dp/Dq system was considered and some universal properties, associated with this critical solution separating the two classes of embedding, were uncovered. In particular it was shown that for a certain temperature the theory exhibits a discrete self-similar behavior, manifested by a double logarithmic spiral in the solution space. This space of solutions is parameterized by the bare quark mass and the fermionic condensate. (Geometrically these correspond, respectively, to the asymptotic separation of the D7- and D3- branes and the degree of bending of the D7-branes away from the D3-branes.)

The region of solution space where the self-similar spiral is located is unstable, in fact: There is a first order phase transition associated with the physics of the system jumping between branches of solutions and bypassing it entirely. Nevertheless, it seems that important features of the full physical story can be captured by examining the neighbourhood of this critical solution. It is remarkable that the critical exponents (or better “scaling exponents”, so as not to confuse the physics with the nomenclature of second order phase transitions) characterizing this logarithmic structure exhibit universal properties and depend only on the dimension of the internal S^n wrapped by the Dq -brane. The precise value of the critical temperature is irrelevant. The structure is determined by focusing on the local geometry near the conical singularity of the critical Dq -brane embedding, and the exponents are then naturally determined by the study of possible embeddings in a Rindler space [7, 9].

The studies described above concern a thermally driven phase transition. As the temperature passes a certain threshold, thermal fluctuations seek out the new global minimum that appears and the system undergoes a transition to a new phase. In this paper we study transitions of the system under the effect of two different types of control parameters: an external electric field and an R-charge chemical potential, revisiting work done on these systems in refs. [16, 18]. We show that the corresponding scaling exponents are again universal and depend only on the dimension of the internal sphere wrapped by the Dq -brane. We find that the key properties of the critical solution can be determined from the local properties of the geometry, and we find that this geometry arises naturally by working in a rotating frame, arrived at using T-duality. The resulting physics is not controlled by thermal dynamics, the local geometry is not Rindler, and so the exponents are different. The phase transition is driven by the quantum (as opposed to thermal) fluctuations of the system, as can be seen from the fact that they persist at zero temperature. It is satisfying that we can cast these different types of transition into the same classifying framework.

The structure of the paper is as follows. We begin in section 2 by reviewing the results

of refs. [7, 9] for the thermally driven phase transition, focusing on the structure of the unstable critical solution, extracting the universal properties of the corresponding scaling exponents. We highlight the natural appearance of a Rindler geometry.

In section 3.1 we consider the case of an external electric field and in the *insulator/conductor* phase transition discussed in ref. [16]. By employing an appropriate T-dual description of the system we demonstrate that the structure of the instability and the scaling exponents can be naturally studied by classifying the possible embedding in a flat *rotating* frame. We observe that these scaling exponents are again universal and depend only on the dimension of the internal S^n sphere, wrapped by the Dq -branes.

In section 3.2 we consider instead the presence of a finite R-charge chemical potential in the Dp/Dq system and demonstrate that the resulting phase transition has the same scaling exponents as the insulator/conductor phase transition driven by an external electric field.

We consider some generalizations of the discussion in section 4 and close with some remarks in section 5.

2. Thermal phase transition

Let us begin by reviewing the result of refs. [7, 9]. We will be using the notations of ref. [9]. Consider the near-horizon black Dp -brane given by:

$$ds^2 = H^{-\frac{1}{2}} \left(-f dt^2 + \sum_{i=1}^p dx_i^2 \right) + H^{\frac{1}{2}} \left(\frac{du^2}{f} + u^2 d\Omega_{8-p}^2 \right), \quad (2.1)$$

$$e^\Phi = g_s H^{(3-p)/4}, \quad C_{01\dots p} = H^{-1},$$

where $H(u) = (R/u)^{7-p}$, $f(u) = 1 - (u_H/u)^{7-p}$ and R is a length scale (the AdS radius in the $p = 3$ case). According to the gauge/gravity correspondence, string theory on this background is dual to a $(p + 1)$ -dimensional gauge theory at finite temperature. Now if we introduce Dq -brane probe having d common space-like directions with the Dp -brane, wrapping an internal $S^n \subset S^{8-p}$ and extended along the holographic coordinate u , we will introduce fundamental matter to the dual gauge theory that propagates along a $(d + 1)$ -dimensional defect.

If we parameterize S^{8-p} by:

$$d\Omega_{8-p}^2 = d\theta^2 + \sin^2 \theta d\Omega_n^2 + \cos^2 \theta d\Omega_{7-p-n}^2, \quad (2.2)$$

where $d\Omega_m^2$ is the metric on a round unit radius m -sphere, the DBI part of the Lagrangian governing the classical embedding of the probe is given by¹:

$$\mathcal{L} \propto e^{-\Phi} \sqrt{-|g_{\alpha\beta}|} = \frac{1}{g_s} u^n \sin^n \theta \sqrt{1 + f u^2 \theta'^2} \quad (2.3)$$

The embeddings split to two classes of different topologies: “Minkowski” embeddings, which have a shrinking S^n above the vanishing locus (the horizon) and yield the physics of

¹We consider only systems T-dual to the D3/D7 one, which imposes the constraint $p - d + n + 1 = 4$.

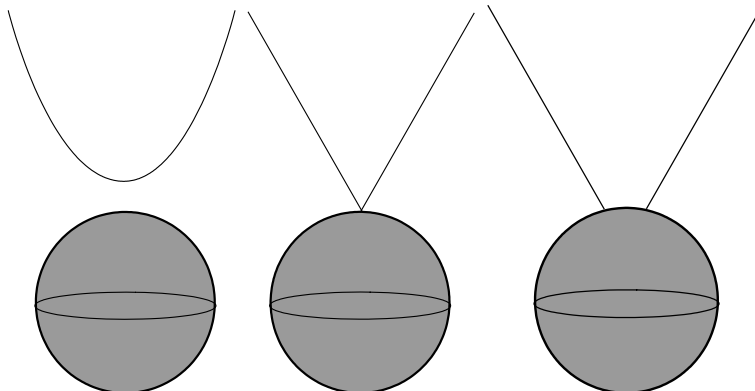


Figure 1: Schematic diagram depicting the Minkowski (left) and black hole (right) embedding solutions that are separated by a “critical” embedding (centre), which has a conical singularity at the event horizon.

meson states and “black hole” embeddings that reach the vanishing locus, corresponding to a melted/deconfined phase of the fundamental matter. These classes are separated by a critical embedding with a conical singularity at the vanishing locus, as depicted in figure 1.

It is convenient to introduce the following coordinates:

$$r^{\frac{7-p}{2}} = \frac{1}{2} \left(u^{\frac{7-p}{2}} + \sqrt{u^{7-p} - u_H^{7-p}} \right), \tag{2.4}$$

$$L = r \cos \theta, \quad \text{and} \quad \rho = r \sin \theta .$$

Then one can show [1, 6] that the asymptotic behavior of the embedding at $\rho \rightarrow \infty$ encodes the bare quark mass $m_q = m/2\pi\alpha'$ and the quark condensate $\langle \bar{\psi}\psi \rangle \propto -c$ of the dual gauge theory via the expansion:

$$L(\rho) = m + \frac{c}{\rho^{n-1}} + \dots \tag{2.5}$$

After solving numerically for each embedding of the Dq -brane, the parameters m and c can be read off at infinity. From the full family of embeddings, a plot of the equation of state of the system $c(m)$ can be generated. The resulting plot for the D3/D7 system [10] is presented in figure 2. The two different colors (and line types) correspond to the two different classes of embeddings. The equation of state is a multi-valued function, and there is a first order phase transition when the free energies of the uppermost and lowermost branches match.

The main subject of our discussion is the spiral structure in the solution space near the critical embedding [9, 14]. In the enlarged portion on the right in figure 2 it is located to the lower left, roughly at $m = 0.9185$, $-c = 0.0225$. The spiral structure that is hidden near this point is a signal of the discrete self-similarity of the theory near the critical solution.

In order to understand the origins of the spiral, we zoom into the space-time region near the tip of the cone of the critical embedding [7, 9] using the change of variables:

$$u = u_H + \pi T z^2 ; \quad \theta = \frac{y}{R} \left(\frac{u_H}{R} \right)^{\frac{3-p}{4}} ; \quad \hat{x} = x \left(\frac{u_H}{R} \right)^{\frac{7-p}{4}} . \tag{2.6}$$

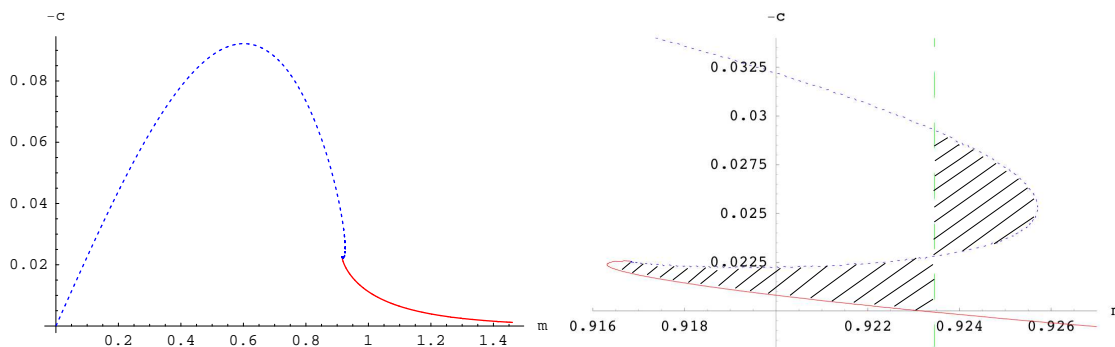


Figure 2: Plot of the equation of state $c(m)$. The zoomed region shows the location of the first order phase transition. There is a spiral structure hidden near the “critical” solution in the neighbourhood of $m = 0.9185$, $-c = 0.0225$

Here T is the temperature of the background given by:

$$T = \frac{7-p}{4\pi R} \left(\frac{u_H}{R} \right)^{\frac{5-p}{2}}. \quad (2.7)$$

Leaving only the leading terms in z results in the following metric:

$$ds^2 = -(2\pi T)^2 z^2 dt^2 + dz^2 + dy^2 + y^2 d\Omega_n^2 + d\hat{x}_d^2 + \dots \quad (2.8)$$

The metric (2.8) corresponds to flat space in Rindler coordinates. The embeddings of the D q -branes in the background (2.8) again split into two different classes: Minkowski embeddings characterized by shrinking S^n ($y = 0$) at some finite z_0 , and black hole embeddings, which reach the horizon at $z = 0$ for some finite $y = y_0$ (the radius of the induced horizon). The equation of motion is derived from the Dirac-Born-Infeld action of the D q -branes, which has the following Lagrangian:

$$\mathcal{L} \propto zy^n \sqrt{1 + y'^2}. \quad (2.9)$$

The equation of motion derived from this reads:

$$zyy'' + (yy' - nz)(1 + y'^2) = 0. \quad (2.10)$$

Solutions of this equation enjoy the scaling property $y(z) \rightarrow \frac{1}{\mu}y(\mu z)$, in the sense that if $y(z)$ is a solution to the equation (2.10) so is $\frac{1}{\mu}y(\mu z)$. Under such a re-scaling the initial conditions (z_0, y_0) for the two classes of embeddings scale as:

$$z_0 \rightarrow z_0/\mu; \quad y_0 \rightarrow y_0/\mu; \quad (2.11)$$

This suggests the existence of a critical solution characterized by $z_0 = y_0 = 0$. One can check that $y = \sqrt{nz}$ is the critical solution. It has a conical singularity at $y = z = 0$.

To analyze the parameter space of the solutions we can linearize the equation of motion (2.10) near the critical solution by substituting $y(z) = \sqrt{nz} + \xi(z)$, for small $\xi(z)$. The resulting equation of motion is:

$$z^2 \xi''(z) + (n+1)(z\xi'(z) + \xi(z)) = 0, \quad (2.12)$$

which has a general solution of the form:

$$\xi(z) = \frac{1}{z^{r_n}} (A \cos(\alpha_n \ln z) + B \sin(\alpha_n \ln z)), \quad (2.13)$$

with $r_n = \frac{n}{2}$; $\alpha_n = \frac{1}{2} \sqrt{4(n+1) - n^2}$.

Note that α_n are real only for $n \leq 4$, which are the cases naturally realized in string theory [14]. Now the scaling property of equation (2.10), combined with the form of the solutions (2.13) suggests the following transformation of the parameters (A, B) under the re-scaling of the initial conditions given in equation (2.11):

$$\begin{pmatrix} A' \\ B' \end{pmatrix} = \frac{1}{\mu^{r_n+1}} \begin{pmatrix} \cos(\alpha_n \ln \mu) & \sin(\alpha_n \ln \mu) \\ -\sin(\alpha_n \ln \mu) & \cos(\alpha_n \ln \mu) \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}. \quad (2.14)$$

For a fixed choice of the parameters A and B , the parameters (A', B') describe a double spiral, whose step and periodicity are set by the real and imaginary parts of the critical/scaling exponents r_n and α_n .

Equation (2.10) has a Z_2 symmetry [7] relating the two classes of solutions (Minkowski and black hole embeddings). If the parameters (A, B) describe one class of embeddings, then the parameters $(-A, -B)$ describe the other. In this way the full parameter space near the critical solution (given by $A = 0, B = 0$) is a double logarithmic spiral.

This self-similar structure of the embeddings near the critical solution in our Rindler space is transferred by a linear transformation to the structure of the solutions in the (m, c) parameter space. If we call (m^*, c^*) the parameters corresponding to the critical embedding from figure 1, then sufficiently close to the critical embedding we can expand:

$$\begin{pmatrix} m - m^* \\ c - c^* \end{pmatrix} = M \begin{pmatrix} A \\ B \end{pmatrix} + O(A^2) + O(B^2) + O(A, B). \quad (2.15)$$

The constant matrix M cannot be determined analytically and depends on the properties of the system. Generically it should be invertible (numerically we have verified that it is) and therefore in the vicinity of the parameter space close to the critical embedding (m^*, c^*) there is a discrete self-similar structure determined by the transformation:

$$\begin{pmatrix} m' - m^* \\ c' - c^* \end{pmatrix} = \frac{1}{\mu^{r_n+1}} M \begin{pmatrix} \cos(\alpha_n \ln \mu) & \sin(\alpha_n \ln \mu) \\ -\sin(\alpha_n \ln \mu) & \cos(\alpha_n \ln \mu) \end{pmatrix} M^{-1} \begin{pmatrix} m - m^* \\ c - c^* \end{pmatrix}. \quad (2.16)$$

Let us define two solutions to be “similar” if:

$$\begin{pmatrix} |m' - m^*| \\ |c' - c^*| \end{pmatrix} = \frac{1}{\mu^{r_n+1}} \begin{pmatrix} |m - m^*| \\ |c - c^*| \end{pmatrix}. \quad (2.17)$$

Then one can see from equation (2.16) that this is possible only for a discrete set of μ s given by:

$$\mu = e^{k\pi/\alpha_n}; \quad k = 1, 2, \dots \quad (2.18)$$

Note that in general the matrix M in equation (2.16) will deform the spiral structure given by the transformation (2.14). However the scaling properties of the theory remain the same as they are completely determined by the scaling exponents: r_n, α_n . Furthermore one can see that the scaling exponents depend only on the dimension of the internal sphere S^n wrapped by the Dq -brane and are thus universal, in the sense that the detailed value of the critical temperature is irrelevant. It is the spiral structure that ultimately seeds the multi-valuedness of the space of solutions, twisting the $(m, -c)$ curve back on itself as in figure 2. Therefore, it is the spiral — and the neighbourhood of the critical solution from where it emanates — that is responsible for the presence of a first order phase transition in the system. Whether there is a spiral or not can be read off from the scaling parameters (r_n, α_n) , and since [14] for all consistent Dp/Dq systems the condition $n \leq 4$ is satisfied the corresponding thermal phase transition (meson melting at large N_c) is a first order one.

3. Quantum-induced phase transitions

In this section we will consider a different class of phase transitions. These arise in the presence of external fields, and can happen even at zero temperature, and so since the fluctuations driving the transition are no longer thermal, they might be expected to be in a different class. Naively, the broad features of the equation of state — multi-valuedness and so forth — have similarities with the thermal case, and so it is natural to attempt to trace the extent to which these similarities persist. We will find that once we cast these systems in the language of the previous section, the similarities and differences will be quite clear.

We will first concentrate on the case of an external electric field. The flavoured system, at large enough electric field, has an *insulator/conductor* phase transition, as studied in ref. [16]. As with the thermal transition of the last section, the mesons dissolve into their constituent quarks, but this time it is due to the electric field overcoming their binding energy. The transition is of first order.

As we saw in the previous section the scaling properties of the thermally driven phase transition are naturally studied in a Rindler frame with a temperature set by the temperature of the background. In ref. [16] it was shown that in analogy to the thermally driven phase transition there is a nice geometrical description of the electrically driven phase transition, and the structure of the system can be again characterized by an unstable critical embedding with a conical singularity at an appropriate vanishing locus (analogous to the event horizon). Here, we will generalize this description to the case of the Dp/Dq system.

Furthermore after an appropriate T-duality transformation we will show that the vanishing locus corresponds to an effective “ergosphere” due to a rotation of the coordinate frame along the compact directions of the background. The instability near criticality is then naturally interpreted as an instability due to the over-spinning of the $D(q - 1)$ brane probes (in the T-dual background) as they reach the ergosphere. We then study the structure of the theory near criticality by zooming in on the space-time region in the vicinity of the conical singularity. Once again, we will find that the structure is entirely controlled by the dimension of the internal sphere, S^n , wrapped by the $D(q - 1)$ -branes (in the T-dual

background) — details such as the value of the electric field and the temperature of the system, are irrelevant.

3.1 Criticality and scaling in an external electric field

Let us consider the near-horizon black Dp -brane given by the background in equation (2.1). Following a similar idea [12] for producing a background magnetic field, if we turn on a pure gauge B -field in the (t, x_p) plane [13, 16, 17], in the dual gauge theory this will correspond to an external electric field, oriented along the x_p direction:

$$B = E dt \wedge dx_p . \tag{3.1}$$

The resulting Lagrangian is:

$$\mathcal{L} \propto e^{-\Phi} \sqrt{-|g_{\alpha\beta} + B_{\alpha\beta}|} = \frac{1}{g_s} \sqrt{\frac{f - E^2 H}{f}} u^n \sin^n \theta \sqrt{1 + f u^2 \theta'^2} . \tag{3.2}$$

This leads to the existence of a vanishing locus at $u = u_*$ given by:

$$u_*^{7-p} = u_H^{7-p} + E^2 R^{7-p} , \tag{3.3}$$

at which the action (3.2) vanishes. Notice that this is distinct from the horizon, and even at zero temperature will be present. A study of the local physics near this locus will therefore pertain to non-thermal physics.

The embeddings split into two different classes: Minkowski embeddings which have a shrinking S^n above the vanishing locus and correspond to meson states and embeddings reaching the vanishing locus, corresponding to a deconfined phase of the fundamental matter. These classes are separated by a critical embedding with a conical singularity at the vanishing locus. Our goal is to explore the self-similar behavior of the theory near this critical embedding and calculate the corresponding scaling exponents.

In order to make the analysis closer to the one performed in refs. [7, 9], for the thermal phase transition (described in the last section), we T-dualize along the x_p direction. This is equivalent to a trading of the pure gauge B -field for a rotating frame in the T-dual background. Indeed the geometry T-dual to equation (2.1), with the B -field given by equation (3.1), is given by:

$$\begin{aligned} d\tilde{s}^2 &= H^{-\frac{1}{2}} \left(-\tilde{f} dt^2 + \sum_{i=1}^{p-1} dx_i^2 \right) + 2H^{\frac{1}{2}} E dt d\tilde{x}_p + H^{\frac{1}{2}} \left(\frac{du^2}{f} + u^2 d\Omega_{8-p}^2 + d\tilde{x}_p^2 \right) , \tag{3.4} \\ e^{\tilde{\Phi}} &= g_s H^{1-\frac{p}{4}} ; \\ \tilde{f} &= 1 - \left(\frac{u_*}{u} \right)^{7-p} . \end{aligned}$$

The background given by equation (3.4) corresponds to the near-horizon limit of a stack of N_c $D(p-1)$ -branes smeared along the coordinate \tilde{x}_p . Now if we place a probe $D(q-1)$ -brane having $(d-1)$ spatial directions shared with the $D(p-1)$ -branes, filling the radial direction u and wrapping an internal S^n inside the S^{8-p} sphere of the background, we will recover the action (3.2), as we should.

Note that in these coordinates we have an effective “ergosphere” coinciding with the vanishing locus given by equation (3.3). Now the critical embedding is the one touching the ergosphere and having a conical singularity at $u = u_*$. In the (m, c) -plane this embedding corresponds to the center of the spiral structure (m_*, c_*) .

Despite the analogy with the analysis of the thermal phase transition, in this case there is a crucial difference, because of the necessity (from charge conservation) for the $D(q - 1)$ -brane to extend beyond the ergosphere. Indeed since the $D(q - 1)$ -brane is an extended object one can find static solutions that extend beyond the ergosphere and are non-superluminal. To this end one should allow the $D(q - 1)$ -brane to extend along the direction of rotation \tilde{x}_p . In the original coordinates (before T-dualization) this is equivalent to a non-trivial profile for the A_p component of the gauge field, which corresponds to the appearance of a global electric current along the x_p -direction [13, 16]. This is the reason why we refer to the corresponding phase transition as an insulator/conductor phase transition. After the transition, the quarks are free to flow under the influence of the electric field, forming a current.

Let us describe how this procedure works in the case of a general $D(p - 1)/D(q - 1)$ -intersection. Again we will work in the T-dual background (3.4). Let us consider an ansatz for the $D(q - 1)$ -brane embedding of the form:

$$\theta = \theta(u) ; \quad \tilde{x}_p = \tilde{x}_p(u) ; \tag{3.5}$$

this leads to the action:

$$\mathcal{L}_* \propto \frac{1}{g_s} \sqrt{\frac{f - E^2 H}{f}} u^n \sin^n \theta \sqrt{1 + f u^2 \theta'^2 + \frac{f^2}{f - E^2 H} \tilde{x}_p'^2} . \tag{3.6}$$

Now after integrating the equation of motion for \tilde{x}_p and plugging the result in the original Lagrangian, we get the following on-shell Lagrangian:

$$\mathcal{L}_* \propto \frac{1}{g_s} \sqrt{\frac{f - E^2 H}{f u^{2n} \sin^{2n} \theta - K^2}} u^{2n} \sin^{2n} \theta \sqrt{1 + f u^2 \theta'^2} . \tag{3.7}$$

It is easy to verify that if we choose the integration constant K^2 in equation (3.7) to satisfy:

$$K^2 = E^2 H_* u_*^{2n} \sin^{2n} \theta_0 , \tag{3.8}$$

then the action (3.7) is regular at the ergosphere ($u = u_*$). Note that at the critical embedding $\theta_0 = \theta_* \equiv 0$ and the constant in equation (3.8) is zero. This constant is proportional to the global electric current along the x_p direction of the original Dp/Dq -brane system. (See refs. [13, 16] for a discussion in the case of the D3/D7 system.)

We are interested in the scaling properties of the theory, near the critical embedding solution. Despite the fact that the Lagrangians (3.2) and (3.7) describing the Minkowski and ergosphere classes of embeddings are different, the fact that at the critical embedding they coincide ($K^2=0$) shows that the corresponding equations of motion share the same critical solution. Furthermore, as we will see, the critical exponents are the same for both types of embedding.

Let us introduce dimensionless coordinates by the transformation:

$$u = u_* + z \frac{Du_*}{7-p}; \quad \theta = \frac{y}{R} \left(\frac{u_*}{R} \right)^{\frac{3-p}{4}}; \quad x_i \left(\frac{u_*}{R} \right)^{\frac{7-p}{4}} \rightarrow x_i; \quad t \left(\frac{u_*}{R} \right)^{\frac{7-p}{4}} \rightarrow t; \quad H_*^{\frac{3}{4}} E \tilde{x}_p \rightarrow \tilde{x}_p, \quad (3.9)$$

where $D^2 = (7-p)^2 f_*/H_*^{\frac{1}{2}} u_*^2$, $H_* = H|_{u=u_*}$ and $f_* = f|_{u=u_*}$. To leading order in z and y the metric (3.4) is given by:

$$d\tilde{s}^2 = -Dzdt^2 + dz^2 + dy^2 + y^2 d\Omega_n^2 + H_*^{\frac{1}{2}} u_*^2 d\Omega_{7-p-n}^2 + 2dt d\tilde{x}_p + \frac{1}{E^2 H_*} d\tilde{x}_p^2 + \sum_{i=1}^{p-1} dx_i^2. \quad (3.10)$$

First consider the case of Minkowski embeddings, characterized by a distance z_0 above the ergosphere at which they close ($y = y(z_0) = 0$). The Lagrangian describing the $D(q-1)$ -brane embedding is:

$$\tilde{\mathcal{L}}_* \propto y^n z^{1/2} \sqrt{1+y'^2}, \quad (3.11)$$

The corresponding equation of motion is given by:

$$\partial_z \left(y^n z^{1/2} \frac{y'}{\sqrt{1+y'^2}} \right) - n y^{n-1} z^{1/2} \sqrt{1+y'^2} = 0. \quad (3.12)$$

Equation (3.12) possesses the scaling symmetry:

$$y \rightarrow y/\mu; \quad z \rightarrow z/\mu; \quad (3.13)$$

in the sense that if $y = y(z)$ is a solution to equation (3.12) so is the function $\frac{1}{\mu} y(\mu z)$. Now under the scaling (3.13) the boundary condition for the Minkowski embedding scales as $z_0 \rightarrow z_0/\mu$. This suggests the existence of a limiting critical embedding with $z_0 = 0$, and indeed:

$$y(z) = \sqrt{2nz}, \quad (3.14)$$

is a solution to the equation of motion in equation (3.12). The corresponding $D(q-1)$ -brane has a conical singularity at $y = z = 0$. Now before we linearize equation (3.12) and calculate the critical exponents let us consider the case of the ergosphere class of solutions characterized by the radius of the ergosphere induced on their world-volume. Because of the possibility to extend beyond the ergosphere we should consider the analog of the ansatz from equation (3.5):

$$y = y(z); \quad \tilde{x}_p = \tilde{x}_p(z). \quad (3.15)$$

The corresponding Lagrangian is:

$$\tilde{\mathcal{L}}_* \propto y^n \sqrt{Dz(1+y'^2) + (Fz+1)\tilde{x}_p'^2}, \quad (3.16)$$

where $F = D/E^2 H_*$. After integrating the equation of motion for \tilde{x}_p and substituting it into the Lagrangian (3.16), we obtain the following on-shell Lagrangian:

$$\tilde{\mathcal{L}}_* \propto \frac{z^{1/2} \sqrt{Fz+1} y^{2n}}{\sqrt{(Fz+1)y^{2n} - y_0^{2n}}} \sqrt{1+y'^2}. \quad (3.17)$$

It is easy to see that the Lagrangian (3.17) is regular at $z = 0, y = y_0$. The equation of motion for $y(z)$, derived from the Lagrangian (3.16) and with the substituted solution for $\tilde{x}_p(z)$ is:

$$\frac{\partial}{\partial z} \left(\frac{z^{1/2} y'}{\sqrt{1+y'^2}} \sqrt{\frac{(Fz+1)y^{2n} - y_0^{2n}}{Fz+1}} \right) - n y^{2n-1} z^{1/2} \sqrt{\frac{Fz+1}{(Fz+1)y^{2n} - y_0^{2n}}} \sqrt{1+y'^2} = 0. \quad (3.18)$$

It is easy to check that equation (3.14) is a solution to equation (3.18). Furthermore for $z \ll 1/F$ one can see that equation (3.18) has the scaling symmetry (3.13) (note that equation (3.13) suggests $y_0 \rightarrow y_0/\mu$). Linearizing equations (3.12) and (3.18) near the critical solution (3.14) by substituting:

$$y(z) = \sqrt{2n}z + \xi(z) \quad (3.19)$$

results in the same equation:

$$z^2 \xi''(z) + (n + 1/2)(z\xi'(z) + \xi(z)) = 0. \quad (3.20)$$

The general solution of equation (3.20) is given by:

$$\xi(z) = \frac{1}{z^{r_n}} (A \cos(\alpha_n \ln z) + B \sin(\alpha_n \ln z)), \quad (3.21)$$

where the scaling exponents are given by:

$$r_n = \frac{2n-1}{4}; \quad \alpha_n = \frac{1}{4} \sqrt{7+20n-4n^2}. \quad (3.22)$$

Note that the scaling exponents again, while quite different from those of the thermal case (see equation (2.13)) depend only on the dimension of the internal S^n wrapped by the Dq-brane and are thus universal for all Dp/Dq systems. Furthermore the discrete self-similarity holds for $n \leq 5$. By similar reasoning to the thermal case [14], since for all consistent systems realized in string theory we have that $n \leq 4$, for such systems we may expect that the electrically driven confinement/deconfinement phase transition is first order and has the described discrete self-similar behavior near the solution that seeds the multi-valuedness of the equation of state.

The rest of the analysis is completely analogous to the thermal case considered in the previous section. Therefore we come to the conclusion that close to the critical embedding (specified by m_* and c_*) the theory has the following scaling property:

$$\begin{pmatrix} m' - m^* \\ c' - c^* \end{pmatrix} = \frac{1}{\mu^{r_n+1}} M \begin{pmatrix} \cos(\alpha_n \ln \mu) & \sin(\alpha_n \ln \mu) \\ -\sin(\alpha_n \ln \mu) & \cos(\alpha_n \ln \mu) \end{pmatrix} M^{-1} \begin{pmatrix} m - m^* \\ c - c^* \end{pmatrix}, \quad (3.23)$$

with r_n and α_n given by equation (3.22).

It is interesting to compare the analytic results some numerical studies. Let us consider the D3/D7 system. From equation (3.13) one can see that the variation of the scaling parameter μ in equation (3.23) can be traded for the variation of the boundary conditions

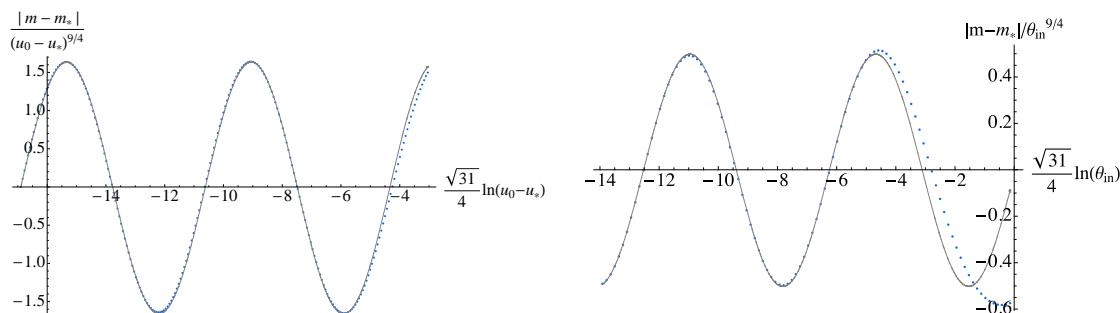


Figure 3: The solid line is a fit with trigonometric functions of period 2π . The plots confirm that the critical exponents of the theory are $r_3 = 5/4$ and $\alpha_3 = \sqrt{31}/4$.

of the probe, namely z_0 for Minkowski and y_0 for ergosphere embeddings. On the other hand, close to the critical embedding, the change of coordinates in equation (3.9) suggests that:

$$\theta_0 \propto y_0 \quad \text{and} \quad u_0 - u_* \propto z_0, \tag{3.24}$$

where u_0 and θ_0 are the boundary conditions for the embeddings in the original (not zoomed in) background. Note that the parameter u_0 is related to the constituent quark mass M_c [14] (in the absence of an electric field) via $M_c = (u_0 - u_H)/(2\pi\alpha')$.

Close to the critical embedding we have that:

$$\mu = (u_{0,\text{in}} - u_*) / (u_0 - u_*) \quad \text{and} \quad \mu = z_{0,\text{in}} / z_0, \tag{3.25}$$

for some fixed boundary conditions $u_{0,\text{in}}$ and $\theta_{0,\text{in}}$. Now equation (3.23) suggests that for Minkowski embeddings the plot of $(m - m_*) / (u_0 - u_*)^{r_n+1}$ versus $\alpha_n \ln(u_0 - u_*)$ should be an harmonic function of $\alpha_n \ln(u_0 - u_*)$ with a period 2π . Similarly for ergosphere embeddings the plot of $(m - m_*) / \theta_0^{r_n+1}$ versus $\alpha_n \ln \theta_0$ should be a harmonic function of $\alpha_n \ln \theta_0$ with a period 2π . Note that the physical meaning of θ_0 can be related to the value of the global electric current (see equation (3.8) and the comment below).

As can be seen in figure 3, for both types of embeddings the numerical results are in accord with equation (3.23) and the analytic results improve deeper into the spiral (large negative values on the horizontal axis). Our numerical results confirm that the critical exponents are indeed $r_3 = 5/4$ and $\alpha_3 = \sqrt{31}/4$, as the general analytic results yield.

3.2 Criticality and scaling with R-charge chemical potential

Now we study the case when the external parameter is an R-charge chemical potential in the dual gauge theory. We will consider the system discussed in ref. [18], where a D7-brane probe in the spinning D3-brane geometry [31, 32] was considered.

The relevant geometry is given by:

$$ds^2 = \Delta^{1/2} \left(-(\mathcal{H}_1 \mathcal{H}_2 \mathcal{H}_3)^{-1} f dt^2 + \frac{u^2}{R^2} d\vec{x}^2 + f^{-1} du^2 \right) + \tag{3.26} \\ + \Delta^{-1/2} \sum_{i=1}^3 \mathcal{H}_i (\mu_i^2 (R d\phi_i - A_i^t dt)^2 + R^2 d\mu_i^2),$$

where

$$f = \frac{u^2}{R^2} \mathcal{H}_1 \mathcal{H}_2 \mathcal{H}_3 - \frac{u_H^4}{u^2 R^2}, \quad \mathcal{H}_i = 1 + \frac{q_i^2}{u^2}, \quad A_t^i = \frac{u_H^2}{R} \frac{q_i}{u^2 + q_i^2},$$

$$\Delta = \mathcal{H}_1 \mathcal{H}_2 \mathcal{H}_3 \sum_{i=1}^3 \frac{\mu_i^2}{\mathcal{H}_i},$$

with $\mu_1 = \sin \theta, \quad \mu_2 = \cos \theta \sin \psi, \quad \mu_3 = \cos \theta \cos \psi.$ (3.27)

Here the parameter u_H would be the radius of the event horizon if the angular momentum of the geometry was set to zero ($q_i = 0$). The radius u_E of the actual event horizon is determined by the largest root of $f(u) = 0$. The temperature of the background is given by [33]:

$$T = \frac{u_E}{2\pi R^2 u_H^2} \left(2u_E^2 + q_1^2 + q_2^2 + q_3^2 - \frac{q_1^2 q_2^2 q_3^2}{u_E^4} \right) = \frac{1}{2\pi R^2 u_H^2 u_E} (u_E^2 - u_1^2)(u_E^2 - u_2^2), \quad (3.28)$$

where u_1 and u_2 are the other two roots of $f(u) = 0$.

The background (3.26) has an ergosphere determined by the expression:

$$\Delta (\mathcal{H}_1 \mathcal{H}_2 \mathcal{H}_3)^{-1} f - \sum_{i=1}^3 \mathcal{H}_i \mu_i^2 (A_t^i)^2 = 0. \quad (3.29)$$

Since the background (3.26) is asymptotically $\text{AdS}_5 \times S^5$, we can “remove” the ergosphere (3.29), by going to a rotating frame. This is equivalent to gauge shifting A_t^i from (3.27) such that $A_t^i = -\mu_R^i + A_t^i$. The parameters μ_R^i are set by the condition $A_t^i|_{u_E} = 0$ and hence:

$$\mu_R^i = \frac{u_H^2}{R} \frac{q_i}{u_E^2 + q_i^2}. \quad (3.30)$$

From the behaviour at infinity ($u \rightarrow \infty$), it is clear that μ_R^i correspond to the angular velocities of the frame along ϕ_i . In the dual gauge theory these correspond to having time dependent phases of the adjoint complex scalars or equivalently to R-charge chemical potentials for the corresponding scalars [30].

In order to restore some of the symmetry of the metric (3.26), we will consider the case when $q_2 = q_3$. This corresponds to having an S^3 (parameterized by ψ, ϕ_2, ϕ_3) inside the deformed S^5 . Now if we introduce D7-branes filling the AdS-like part of the geometry and wrapping the S^3 , we will add fundamental matter to the gauge theory. Furthermore we are free to rotate the D7-branes along ϕ_1 and the corresponding angular velocity is interpreted as a time dependent phase of the bare quark mass². (Recall that in introducing D7-branes to the D3-brane system we actually add flavours as chiral superfields into the $\mathcal{N} = 2$ gauge theory). If that phase is the same as the phase of the complex adjoint scalar, $\mu_R^1 t$, it is equivalent to a R-charge chemical potential for both the adjoint scalar and the chiral field.

On the gravity side of the description this is equivalent to letting the D7-branes have the same angular velocity μ_R^1 as the rotating frame of the background. Moving to the

²We would like to thank A. Karch for pointing this out to us.

frame co-rotating with the D7-brane corresponds to moving back to the gauge choice for A_t^1 from equation (3.27). The price that we pay is that we again have an ergosphere in the bulk of the background. It will be convenient to gauge shift A_t^2 and A_t^3 to $A_t'^2$ and $A_t'^3$, correspondingly, so that the ergosphere is at:

$$\Delta(\mathcal{H}_1\mathcal{H}_2^2)^{-1}f - \mathcal{H}_1 \sin^2 \theta (A_t^1)^2 = 0. \tag{3.31}$$

The shifted forms, $A_t''^2$ and $A_t''^3$, vanish at the locus given by equation (3.31).

The possible D7-brane embeddings then naturally split into two classes: Minkowski embeddings that have a shrinking S^3 above the ergosphere and ergosphere embeddings which reach the ergosphere. These classes are again separated by a critical embedding which has a conical singularity at the ergosphere. In analogy to the T-dual description of the previous subsection for the external electric field case, the ergosphere embeddings will have to be extended along ϕ_1 so that they can stay non-space-like beyond the ergosphere³. However in this paper we are interested in the scaling properties of the theory for parameters (m, c) in the vicinity of the critical parameters (m_*, c_*) , corresponding to the critical embedding. As we saw in the previous section, modifying the ergosphere class of embeddings so as to be regular at the ergosphere does not alter the properties of the theory near the critical solution. In particular the scaling exponents characterizing the discrete self-similar behavior of the theory remain the same. So henceforth we will focus on the study of the Minkowski type of embeddings. The analysis is completely analogous to the one performed in the previous subsection.

In order to focus on the space-time region close to the conical singularity of the critical embedding, we consider the change of coordinates:

$$u = u_{\text{erg}} + \frac{u_H q_1}{R u_{\text{erg}}} z; \quad \cos \theta = \frac{\pi}{2} - \frac{y}{R}, \tag{3.32}$$

where

$$u_{\text{erg}}^2 = u_H^2 - q_2^2 \tag{3.33}$$

is the radial coordinate u of tip of the critical embedding or equivalently the $\theta = \pi/2$ point of the ergosphere. It can be shown that for the values of q_2 for which the geometry is not over spun (and so has an horizon) the corresponding value of u_{erg} is real.

After leaving only the leading terms in z and y , we get:

$$ds^2/\alpha' = -D_1 z dt^2 + dz^2 + dy^2 + y^2 d\Omega_3^2 - 2q_1 dt d\phi_1 + \frac{u_H^2}{R^2} d\vec{x}^2 + R_1 d\phi_1^2, \tag{3.34}$$

$$d\Omega^3 = d\psi^2 + \sin^2 \psi d\phi_2^2 + \cos^2 \psi d\phi_3^2; \quad D_1 = \frac{4q_1 u_H}{R^3}; \quad R_1^2 = \frac{u_{\text{erg}}^2 + q_1^2}{u_H^2} R^2.$$

The metric in equation (3.34) is of the same type as that in equation (3.10), namely flat space with some compact directions in a rotating frame. Therefore the analysis is completely analogous to the one for the electric case and hence the scaling exponents are

³Note that in [18] the ergosphere class of embeddings are not extended along ϕ_1 .

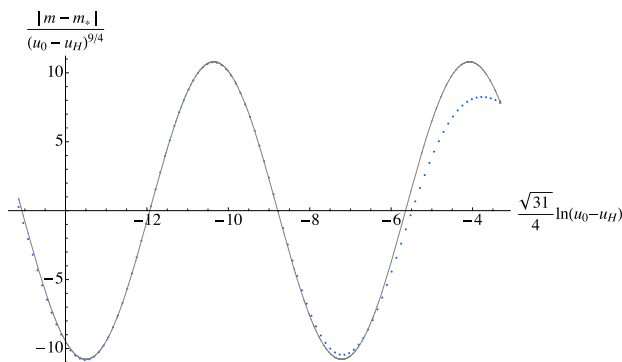


Figure 4: Plot of the relation between the bare quark mass parameter m and the distance above the ergosphere $(u_0 - u_H)$. The plot is for $q = 0.5$ in units in which $u_H = 1$. The solid line is a fit with trigonometric functions of a period 2π . The plot confirms that the scaling exponents of the theory are $r_3 = 5/4$ and $\alpha_3 = \sqrt{31}/4$.

again given by equation (3.22) with $n = 3$, because the D7-branes are wrapping an internal S^3 :

$$r_3 = 5/4 ; \quad \alpha_n = \sqrt{31}/4 . \tag{3.35}$$

We can again verify this numerically. It is convenient to do this for the single charge case, namely $q_1 \neq 0, q_2 = q_3 = 0$. The plot analogous to figure 3 for the electric case, is presented in figure 4. The plot represents the variation of the bare quark mass parameter m as a function of the initial boundary condition $u_0 - u_H$, for Minkowski-type embeddings. The parameter m_* corresponds again to the bare quark mass for the state corresponding to the critical embedding. The good agreement with the result for the critical exponents in equation (3.22) is clear, and the accuracy of the analytic description improves as we go deeper into the spiral (to the left).

An important observation is that our result does not depend on the values of the R-charges, nor the temperature. In fact, this physics persists at zero temperature, such as at extremality with all three charges equal $q_1 = q_2 = q_3 = q$, or more generally. (Extremality is when $u_E = u_1$ or u_2 , for which $T = 0$. See equation (3.28).) The fact that we have the same structure at zero temperature (extremal horizon) further confirms that the key properties of the corresponding phase transition is indeed driven by the quantum (rather than thermal) fluctuations of the system.

4. Criticality and scaling: some generalizations

In this section we generalize the procedure for the study of the critical behavior employed in all three different systems of phase transition (thermal, or in the presence of electric field or R-charge chemical potential). This may lay the groundwork for other types of phase transitions that may arise in future studies, seeded by spirals with different universal behaviour.

Note that in all cases there is some vanishing locus. The different classes of Dq-brane's embeddings are being classified with respect to whether they fall into that vanishing locus,

or wrap an internal S^n sphere that is contracting to zero size above the vanishing locus signaling the end of the Dq-brane.

In all cases there is a critical embedding that separates the two classes of embeddings. The critical embedding reaches the vanishing locus and has a conical singularity there at some finite radius u_* ($u_* = u_H$ or u_{erg} for the thermal and R-charge cases).

The main point of the analysis is that after zooming into the space-time region near the conical singularity we obtain the metric:

$$ds^2 = -Dz^k dt^2 + dz^2 + dy^2 + y^2 d\Omega_n^2 + \dots, \quad (4.1)$$

where D is a non-essential constant. The Dirac-Born-Infeld Lagrangian of the brane is then:

$$\mathcal{L} \propto z^{k/2} y^n \sqrt{1 + y'^2}. \quad (4.2)$$

Note that to extract the key behavior (that we are studying) of this critical embedding (and its neighbourhood) there is no need to modify the embeddings which reach the vanishing locus (as we did for the ergosphere class of embeddings). The critical solution and the linearized equation of motion is the same for both classes. Therefore it is sufficient to consider the Minkowski type of embeddings and analyze the Lagrangian (4.2). The resulting equation of motion is:

$$\partial_z \left(\frac{z^{k/2} y^n y'}{\sqrt{1 + y'^2}} \right) - n y^{n-1} z^{k/2} \sqrt{1 + y'^2} = 0. \quad (4.3)$$

It is easy to check that equation (4.3) has the scaling property (3.13) and the limiting critical solution is given by:

$$y_*(z) = \sqrt{\frac{2n}{k}} z. \quad (4.4)$$

Now after the substitution:

$$y(z) = \sqrt{\frac{2n}{k}} z + \xi(z), \quad (4.5)$$

we obtain the following linearized equation:

$$z^2 \xi''(z) + (n + k/2)(z\xi'(z) + \xi(z)) = 0. \quad (4.6)$$

The general solution of equation (4.6) can be written as:

$$\xi(z) = \frac{1}{z^{r_n^{(k)}}} (A \cos(\alpha_n^{(k)} \ln z) + B \sin(\alpha_n^{(k)} \ln z)), \quad (4.7)$$

where

$$r_n^{(k)} = (n + k/2 - 1)/2; \quad \alpha_n^{(k)} = \frac{1}{2} \sqrt{4(n + k/2) - (n + k/2 - 1)^2}; \quad (4.8)$$

are the scaling exponents characterizing the self-similar behavior of the theory. Both being real, they control the shape of the spiral which emanates from the critical solution. The oscillatory behavior is present for $n \leq 3 + 2\sqrt{2} - k/2$. For these values of n the theory

exhibits a discrete self-similarity and the equation of state $c = c(m)$ is a multi-valued function suggesting that the corresponding phase transition is a first order one.

While there is the possibility of complex scaling exponents and hence possibly second order phase transitions (if the multi-valuedness goes away when the spiral does), this is not realized in the examples that we know from string theory.

Note that we have $k = 2$ for a thermal induced phase transition and $k = 1$ for the quantum induced phase transitions that we studied (external electric field and R-charge chemical potential), arising from the two most natural types of a vanishing locus that one may have: an horizon, and an ergosphere. Perhaps other systems will yield different values of k .

5. Closing remarks

We have succeeded in casting two important types of phase transition (in large N_c gauge theory with fundamental flavours) into the same classifying framework as the meson-melting phase transition. These quantum fluctuation induced transitions (so-called since they persist at zero temperature), resulting in the liberation of quarks from being bound into mesons as a result of the application of an external electric field, or a chemical potential for R-charge, turn out to have the same underlying structure. It is distinct from that found for thermal fluctuation induced transitions. The structures are controlled by the local geometry of the spacetime seen by a critical D-brane embedding (it is the borderline case between two physically distinct classes of embedding), and while it is Rindler for the thermal case with an horizon at the origin, it is (after a T-duality in order to geometrize the discussion as much as possible) a rotating space with a simple “ergosphere” type locus. The technique of characterizing the physics in terms of this underlying classifying space [7, 9] is rather pleasing in its utility, and we extended our analysis to the natural generalization of this space, extracting the scaling exponents that might pertain to physics from future studies.

Of course, there is much interest in how much we can learn about finite N_c physics (for applications to systems such as QCD) by studying universal features of large N_c . Unfortunately, it is almost certain that much of this is far from robust against $1/N_c$ corrections. The spiral structure is rather delicate, and the stringy corrections arising in going away from the large N_c limit would generically severely modify the classifying spacetimes we’ve been studying, erasing the spiral and its self-similarity. The absence of the spiral is necessary for there to be (at best) a second-order transition at finite N_c , since it results in multi-valuedness of the solution space, requiring the system to perform a first order jump.

It is tempting to speculate, however, that the nature by which the spiral is destroyed by $1/N_c$ corrections might (especially since the setting is so geometrical) be characterizable in a way that allows universal properties of the second (or higher) order phase transitions to be deduced from the properties of the spiral at large N_c . We leave such explorations for later work.

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